Permutation and Combination

Module-4

Recap

"Fundamental Principle of Counting states that
"If an event can occur in **m** different ways , following which another event can occur in **n** different ways, then the total number of occurrence of the events in the given order is **mx n** ."

The notation 'n!' represents the product of first n natural numbers

A Permutation is an arrangement in a definite order of number of objects taken some or all at a time

For a natural number 'n'

$n! = n(n-1)\,!$
$= n\,(n-1)\,(n-2)\,!$
$= n(n-1)\,(n-2)\,(n-3)\,!$
....

Important results

$^nP_n = n!$

$^nP_1 = n$

$^nP_0 = 1$

Permutations when **all the objects are distinct**

Permutations when **all the objects are not distinct**

Theorem 1

The number of permutation of n different objects taken r at a time, where 0< r ≤ n and the objects do not repeat is ${}^{n}P_{r}$

Theorem 2

The number of Permutations of n different objects taken r at a time,when repetition is allowed is n^r

Theorem 3

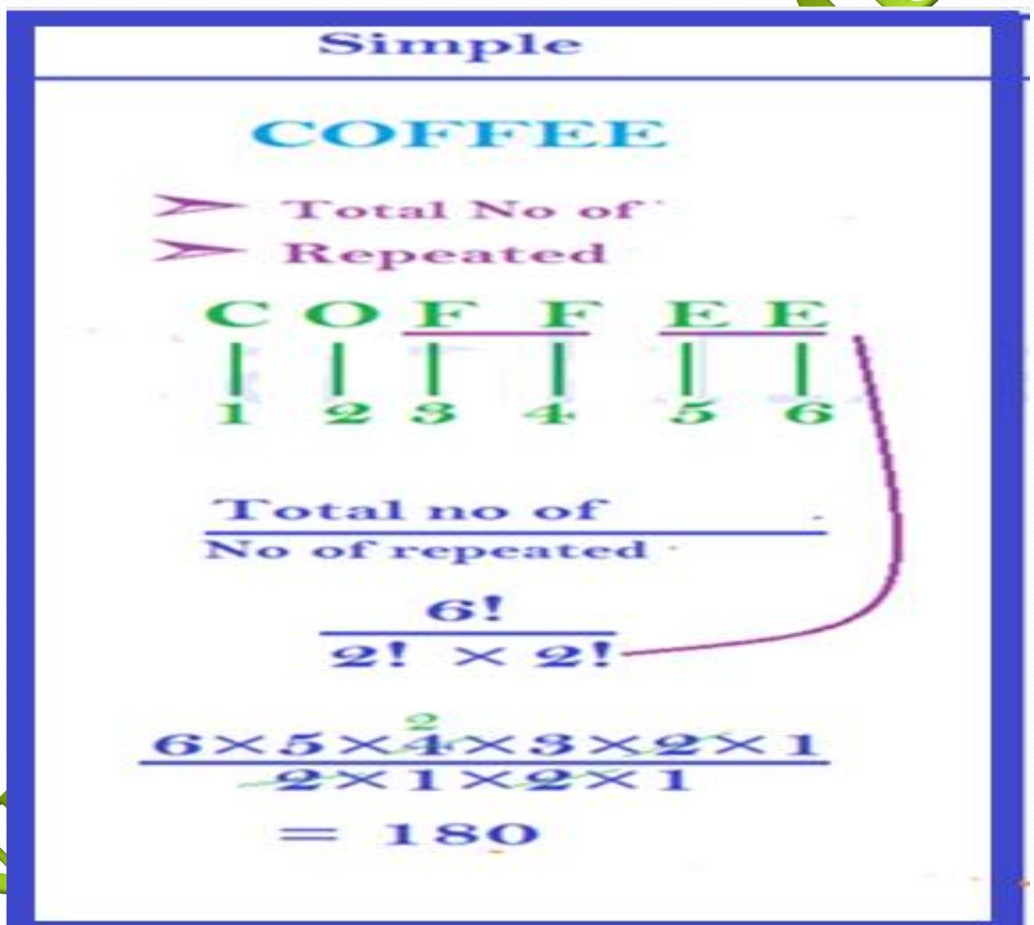
The number of permutations of n objects ,where p objects are of the same kind and the rest are all different = $\frac{n!}{p!}$

Theorem 4

The number of permutations of n objects , where p_1 objects are of one kind , p_2 are of second kind,….p_k are of k^{th} kind and the rest , if any are of different kind is $\frac{n!}{p_1! p_2! \ldots p_k!}$

Find the number of arrangements of the letters of the word COFFEE.

LET'S SOLVE



MALAYALAM

TOTAL =

M comes-

A comes-

L comes-

?????

Question 10:
In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Answer 10:

In the given word MISSISSIPPI, I appears 4 times, S appears 4 times, P appears 2 times, and M appears just once.

$$= \frac{11!}{4!4!2!}$$
$$= \frac{11\times10\times9\times8\times7\times6\times5\times4!}{4!\times4\times3\times2\times1\times2\times1}$$
$$= \frac{11\times10\times9\times8\times7\times6\times5}{4\times3\times2\times1\times2\times1}$$
$$= 34650$$

M	S	S	S	S	P	P	I I I I
1	2	3	4	5	6	7	8

These **8** objects in which there are 4 Ss and 2 Ps can be arranged in $\frac{8!}{4!\,2!}$ ways i.e., 840 ways.

4 I's do not come together = 34650 − 840 = 33810

Permutations continued

In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same color are indistinguishable ?

Sol: Total number of discs are 4 + 3 + 2 = 9. Out of 9 discs, 4 are of the first kind (red), 3 are of the second kind (yellow) and 2 are of the third kind (green).
Thus number of permutation is:

$$\boxed{\frac{9!}{4!3!2!} = 1260}$$

- Find the number of the arrangement of all nine letters of word SELECTION in which the two letters E are not next to each other.
- Solutions:

  Total no. of arrangements – No. of arrangements with two E next to each other

  $$= \frac{9!}{2} - 8!$$

  $$= 141120$$

Find number of arrangements of the letters of the word PENALTY such that vowels come together.

PENALTY

P N L T Y A E

1 2 3 4 5 6

$6! \times 2! = 1440$

P N L T A E Y

P N L T E A Y

P N L A E T Y

P N L E A T Y

Poll

In how many different ways can the letters of the word CORPORATION be arranged so that the vowels always come together?

(A) 810 (B) 1440 (C) 2880 (D) 50400

Permutation Word Problems………

Example

Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,

(i) do the words start with P

12 letters,

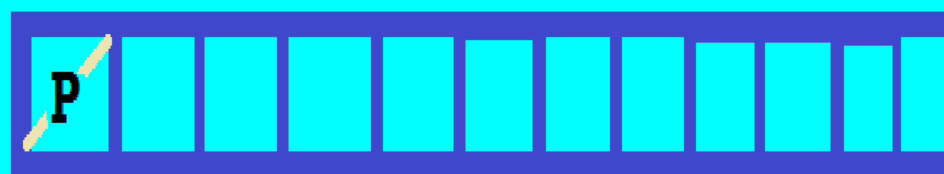
N appears 3 times,

E appears 4

D appears 2 times

The required number of arrangements:

$$\frac{12!}{4!3!2!} = 1663200$$



11 LETTERS: N(3);E(4);D(2)

remaining 11 letters. Therefore, the required number of words starting with P are

$$\frac{11!}{4!3!2!} = 138600$$

(ii) do all the vowels always occur together?

INDEPENDENCE.

There are 5 vowels in the given word, which are 4 Es and 1 I. Since, they have to always occur together, we treat them as a single object EEEEI for the time being. This single object together with 7 remaining objects will account for 8 objects. These 8 objects, in which there are 3Ns and 2 Ds, can be rearranged in

$$\frac{8!}{3!2!} \text{ ways}$$

Corresponding to each of these arrangements, the 5 vowels E, E, E, E and I can be rearranged in

$$\frac{5!}{4!}$$

ways. Therefore, by multiplication principle the required number of arrangements

$$\frac{8!}{3!2!} \times \frac{5!}{4!} = 16800$$

N D P N D N C IEEEE

1 2 3 4 5 6 7 8

Total = 8 : N = 3: D = 2

can be rearranged in $\frac{8!}{3!2!}$ ways

I EE EE

E I E E E

EE I EE

EE E I E

EE EE I

can be rearranged in $\frac{5!}{4!}$ ways

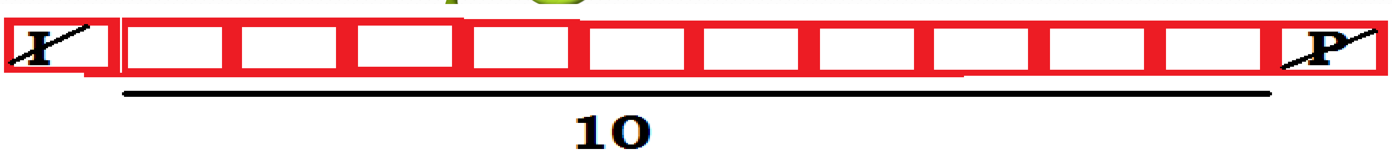
TOTAL no.of arrangements,all vowels together = $\frac{8!}{3!2!} \times \frac{5!}{4!} = 16800$

(iii) do all the vowels never occur together?

(iii) The required number of arrangements = the total number of arrangements (without any restriction) – the number of arrangements where all the vowels occur together.

$$= 1663200 - 16800 = 1646400$$

(iv) do the words begin with I and end in P?



N appears 3 times,
E appears 4
D appears 2 times

(iv) Let us fix I and P at the extreme ends (I at the left end and P at the right end). We are left with 10 letters. Hence, the required number of arrangements

$$\frac{10!}{4!3!2!} = 12600$$

- Example: How many words can be formed with the letters of the word 'OMEGA' when:
- (i) 'O' and 'A' occupying end places.
- (ii) 'E' being always in the middle
- (iii) Vowels occupying odd-places
- (iv) Vowels being never together.
- Ans.
- (i) When 'O' and 'A' occupying end-places
- => M.E.G. (OA)
- Here (OA) are fixed, hence M, E, G can be arranged in 3! ways
- But (O,A) can be arranged themselves is 2! ways.
- => Total number of words = 3! x 2! = 12 ways.

- (ii) When 'E' is fixed in the middle
- => O.M.(E), G.A.
- Hence four-letter O.M.G.A. can be arranged in 4! i.e 24 ways.
- (iii) Three vowels (O,E,A,) can be arranged in the odd-places (1^{st}, 3^{rd} and 5^{th}) = 3! ways.
- And two consonants (M,G,) can be arranged in the even-place (2^{nd}, 4^{th}) = 2 ! ways
- => Total number of ways= 3! x 2! = 12 ways.
- (iv) Total number of words = 5! = 120!
- If all the vowels come together, then we have: (O.E.A.), M,G
- These can be arranged in 3! ways.
- But (O,E.A.) can be arranged themselves in 3! ways.
- => Number of ways, when vowels come-together = 3! x 3!
- = 36 ways
- => Number of ways, when vowels being never-together
- = 120-36 = 84 ways.

Find the number of words with or without meaning which can be made using all the letters of the word AGAIN .If these words are written as in a dictionary , what will be the 50^{th} word?

Solution There are **5** letters in the word **AGAIN**, in which **A** appears **2** times. Therefore, the required number of words = $\frac{5!}{2!}$ =60

To get the number of words starting with A, we fix the letter A at the extreme left position, we then rearrange the remaining 4 letters taken all at a time. There will be as many arrangements of these 4 letters taken 4 at a time as there are permutations of 4 different things taken 4 at a time. Hence, the number of words starting with

A = 4! = **24**. Then, starting with **G**, the number of words = $\frac{4!}{2!}$ = **12** as after placing G at the extreme left position, we are left with the letters A, A, I and N. Similarly, there are **12** words starting with the next letter **I.** Total number of words so far obtained = 24 + 12 + 12 =48.

The **49th** word is **NAAGI**. The **50th** word is **NAAIG**.

$\cancel{A}$	G	A	I	N

4! = **24**.

$\cancel{G}$	A	A	I	N

$\frac{4!}{2!} = 12$

$\cancel{I}$	A	A	G	N

$\frac{4!}{2!} = 12$

TOTAL= 24+12+12=48

WHAT'S 49^{TH} = N - - - -
THEN 50^{TH} = N - - - -

ASSIGNMENT

1	**How many words can be formed out of the letters of the word ' TRIANGLE' ? How many of these will begin with T and end with E?**
2	How many 6-digit numbers can be formed from the digits 0 ,1 , 3 , 5 , 7 and 9 which are divisible by 10 and no digit is repeated ?
3	Find the number of different permutations of the letters of the word BANANA.
4	How many numbers greater than **1000000** can be formed by using the digits **1 , 2 , 0 , 2 , 4 , 2 , 4**?
5	Letters of the word 'MOTHER' are arranged in all possible ways and the words (with or without meaning)so obtained are arranged as in a dictionary. What is the position of the word 'MOTHER' in this arrangement?
	ANSWERS;(1) **8!=40320 and 6! = 720** (2) **120** (3) $\frac{6!}{3!2!}$= **60** (4) **360** (5) **309**th

THANK YOU

Stay safe

Stay blessed